Muon Track Reconstruction and Data Selection Techniques in AMANDA

- Introduction
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- Critique and Outlook

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Detection Modes

Track-like

Cascade-like

PMTs

\( \theta_c \)

muon

Cherenkov cone

cascade

spherical Cherenkov front
Arrival Time

\[
t_{\text{geo}} = t_0 + \frac{\hat{p} \cdot (r_i - r_0) + d \tan \theta_c}{c_{\text{vac}}}
\]
Maximize:
\[ L(\text{Event} = \{t_1, A_1, \ldots, t_n, A_n\} \mid \text{Track} = \{r_0, t_0, p, E\}) \]

Single Photo-Electron Time Likelihood (SPE)

\[ L_{\text{time}} = \prod_{i=1}^{N_{\text{hits}}} p_1(t_{\text{res},i} \mid a = d_i, \eta_i, \ldots) \]
Multi Photo-Electron Likelihood (MPE)

\[
p_{N}^{1}(t_{\text{res}}) = N \cdot p_{1}(t_{\text{res}}) \cdot \left( \int_{t_{\text{res}}}^{\infty} p_{1}(t) dt \right)^{(N-1)} = N \cdot p_{1}(t_{\text{res}}) \cdot (1 - P_{1}(t_{\text{res}}))^{(N-1)}
\]

Poisson Saturated Amplitude Likelihood (PSA)

\[
p_{\mu}^{1}(t_{\text{res}}) = \frac{1}{N} \sum_{i=1}^{\infty} \frac{\mu^{i} e^{-\mu}}{i!} \cdot p_{1}^{i}(t_{\text{res}}) = \frac{\mu}{1 - e^{-\mu}} \cdot p_{1}(t_{\text{res}}) \cdot e^{-\mu} P_{1}(t_{\text{res}})
\]

If several photons arrive, the earliest arrives earlier than predicted by the SPE likelihood.
Likelihood Extensions

**Phit-Pnohit Likelihood**

\[
\mathcal{L}_{\text{hit}} = \prod_{i=1}^{N_{\text{ch}}} P^{\text{hit},i} \cdot \prod_{i=N_{\text{ch}}+1}^{N_{\text{OM}}} P^{\text{no-hit},i}.
\]

essentially evaluates mean visual range

**Combination of Time and Phit Likelihood**

\[
\mathcal{L}_{\text{MPE} \oplus \text{PhitPno-hit}} = \mathcal{L}_{\text{MPE}} \cdot (\mathcal{L}_{\text{hit}})^{w}
\]

Currently best performance
Likelihood of Waveforms

Likelihood of a waveform with $N$ photons (resolved)

$$L = \frac{N!}{\prod_{i=1}^{N} p_1(t_i)}$$

Likelihood of a waveform with $N$ pulses of $npe_i$ photons (unresolved)

$$L = \frac{N!}{\prod_{i=1}^{N} npe_i \cdot p_1(t_i) \cdot (P_1(t+\Delta) - P_1(t_i))^{npe_i-1}}$$
Zenith weighted reconstruction

Maximize:

\[ L(\text{Track} | \text{Event}) = \frac{L(\text{Event} | \text{Track}) \times L(\text{Track})}{L(\text{Event})} \]

Motivated by Baye’s Theorem

Zenith weighted Reconstruction:
\[ L(\text{Track}) = \Phi(\theta) \]

Convergence in one hemisphere
\[ L(\text{Track}) = \Theta(\pm(\theta - 90)) \]
3d-Simulation of photons emitted from track-segments and cascades: Arrival probability and time at the receiver
• Direct reconstruction using PTD tables
  -> extension layered Ptd and Photonics
• Simplified analytical parameterization with a Gamma distribution (Pandel function)

\[
p(t_{\text{res}}) \equiv \frac{1}{N(d)} \frac{\tau^{-(d/\lambda)} \cdot t_{\text{res}}^{(d/\lambda-1)}}{\Gamma(d/\lambda)} \cdot e^{-\left(t_{\text{res}} \cdot \left(\frac{1}{\tau} + \frac{c_{\text{medium}}}{\lambda_a}\right) + \frac{d}{\lambda_a}\right)}
\]

\[
N(d) = e^{-d/\lambda_a} \cdot \left(1 + \frac{\tau \cdot c_{\text{medium}}}{\lambda_a}\right)^{-d/\lambda}
\]

Very few parameters describe the full phase space:

\[
\begin{align*}
\tau &= 557 \text{ ns} \\
\lambda &= 33.3 \text{ m} \\
\lambda_a &= 98 \text{ m}
\end{align*}
\]

\[
\begin{align*}
d_{\text{eff}} &= a_0 + a_1 \cdot d \\
a_1 &= 0.84 \quad a_0 = 3.1 \text{ m} - 3.9 \text{ m} \cdot \cos(\eta) + 4.6 \text{ m} \cdot \cos^2(\eta)
\end{align*}
\]

Fast calculation, integrable (->MPE ...)
Limited accuracy, need to convolute with a Gaussian (PMT jitter)

Only Bulk ice (status 1998) implemented. Upgrade is underway!
Parameterization

\( d = 8 \text{ m} \)

\( d = 71 \text{ m} \)

- **Ptd bulk ice (1998)**
- **Pandel parameterization**
Assume all hits are points on a line, which is moving with velocity $v$ through the detector

$$\mathbf{r}_i \approx \mathbf{r} + \mathbf{v} \cdot t_i$$

Constructing:

$$\chi^2 \equiv \sum_{i=1}^{N_{\text{hit}}} (\mathbf{r}_i - \mathbf{r} - \mathbf{v} \cdot t_i)^2$$

$$d\chi^2 / dr = 0$$ and $$d\chi^2 / dv = 0$$ gives the analytic solution:

$$\mathbf{r} = \langle \mathbf{r}_i \rangle - \mathbf{v} \cdot \langle t_i \rangle \quad \mathbf{v} = \frac{\langle \mathbf{r}_i \cdot t_i \rangle - \langle \mathbf{r}_i \rangle \cdot \langle t_i \rangle}{\langle t_i^2 \rangle - \langle t_i \rangle^2}$$

V. Stenger, J. Jacobsen, A. Roberts (?)
Initial Track: Direct Walk

Four step fast pattern recognition algorithm:

1.) **Select track-elements (TE)** by finding distant OMs (d>50m) for which:
   \[ |\Delta t| < \frac{d}{c_{vac}} + 30\text{ns}, \]  
   (causality)

2.) **Select associated hits (AH)** for each TE with reasonable relative times:
   \[-30\text{ns} < t_{res} < 300\text{ns}, \quad d < 25m \text{ ns}^{1/4} / (t_{res}+30\text{ns})^{1/4}\]

3.) **Select track candidates (TC):**
   More than 10 AH and lever arm (RMS of AH points) > 20m

4.) **Cluster search:** Select cluster with most TC
   \[ \psi < 15^\circ, \quad Q_{TC} > 0.7 \cdot Q_{max} \quad \text{with} \quad Q_{TC} = \min(N_{AH}, 0.3m^{-1} \cdot \sigma_L + 7) \]

Algorithm is **fast, efficient** and has a **good angular resolution.**

**Capable to identify muon bundles**

(P. Steffen, 2001)
Minimization of $-\log(L)$
- Powell’s (NR)
- Minuit (Mini)
- Simplex
- Simulated annealing (NR)
- TMinuit

Problems:
- Local Minima
- Vertical coordinates (ambiguity in azimuth)

Manipulate initial track to improve convergence:
- Shift $r_0$ close to COG
- Transform $t_0$: Evaluate all arrival times and avoid negative times

$\rightarrow$ Iterative Reconstruction
Iterative Minimization

Ability to find global minimum depends on initial track but 5 free parameter ($\theta$, $\varphi$, $r_0$) -> no systematic scan of the full parameter space

Iterate: Reconstruct the event N - times.
  Use the track result of previous iteration and randomize $\theta$ and $\varphi$
  Use reasonable values for $r_0$ (shift to COG, $t_0$ shift)

Store each found minimum and use finally the best result

Already about 10 to 20 iterations are sufficient to find the global minimum.

CPU time is proportional N but ok for filtered data
3 (+1) Strategies:

- **Fit mean visual range with** $P_{\text{hit}} - P_{\text{nohit}}$ Likelihood
  - Amplitude Likelihood
    - Complicated, needs detailed Ice Model
    - Fluctuations, Dynamic range
  - Neural Network
    - use energy correlated variables
    - presently only simple variables $N\text{ch}, N\text{hits}, \langle \text{ADC} \rangle, \langle \text{LE} \rangle, \text{RMS(LE)}$ ...
    - Use reconstruction geometry information in future
- **Cut on Nch** ($\rightarrow$ diffuse limit)
Neural Network result

MC Test network with mono-energetic muons

Result after unfolding the energy spectrum of measured neutrinos (2000 point-source sample)
Background classes:

- Horizontal muons
- Muon bundles
- Secondary cascades (brems)
- Stopping muons
- Scattering ice layers
- Corner clippers
- Synchronous muons
- Instrumental effects (X-talk, noisy channels)

Important selection cuts:

\[ l = -\log(L)/N_{\text{free}} \]
\[ l_{\text{track}} / l_{\text{cascade}} \]
\[ l_{\text{up}} / l_{\text{down}} \]
\[ N_{\text{direct}} : \text{Number of unscattered hits} \]
\[ L_{\text{direct}} : \text{Track length (Lever arm)} \]
\[ S : \text{Smoothness} = \text{MAX}(S_j) \]

\[
S_j \equiv \frac{j - 1}{N - 1} - \frac{l_j}{l_N}
\]

\[
S_j^{\text{hit}} \equiv \frac{\sum_{i=1}^{j} A_i}{\sum_{i=1}^{N_{\text{OM}}} A_i} - \frac{\sum_{i=1}^{j} P_{\text{hit},i}}{\sum_{i=1}^{N_{\text{OM}}} P_{\text{hit},i}}
\]

Typically these cuts quantify information which was not evaluated in the reconstruction itself... many more
Strawman analysis for the demonstration of the typical AMANDA-II performance:

Cuts: \( N_{ch}, N_{dir}, L_{dir}, l_{SPE}, \psi \) (DW, SPE, MPE)

Cut level:
Each cut set to 95% passing efficiency for atm. neutrinos relative to the previous level

- Angular resolution strongly depends on the selection
- AMANDA-II achieves an angular resolution of typically 2°

=> Use “Cut level 6” sample in the following
Zenith angle dependence due to geometry of the AMANDA-II detector
Small zenith shift (± 0.5°) as function of the zenith angle due to geometry of the AMANDA-II detector

- Verified by SPASE coincident events
- Can be corrected for (Not corrected here)
Angular resolution is degrading at higher energies because of a wrong fit model:

- SPE fit
- infinite minimum ionizing track (but secondary cascades dominate the light output)

-> improved likelihood model
-> pattern recognition (in wave-forms)
The reconstruction is insensitive to calibration uncertainties, which are of the order of 5-7 ns.
Current performance estimates of IceCube are only based on the standard AMANDA reconstruction.

Still a lot of things improve:
- $\Delta \psi \sim 0.6^\circ - 0.8^\circ \ (E > 1 \text{TeV})$
- no degradation at high energies
- good performance at horizon

We expect a strong improvement, after establishing advanced capabilities of the IceCube Digital OMs.
Critique of the current reconstruction

Reconstruction in AMANDA is still (!) not final. So far the collaboration concentrated on:
Finding a method that works and producing first physics results.
(so, its actually bad and that's why we can expect strong improvements in the future)

The current Likelihood Model: The assumption of a single minimum ionizing track is an underlying problem when trying to improve the current performance.

The current Parameterization:
• The optical model is completely outdated (Old values and no vertical ice properties)
• The Pandel parameterization is not very accurate.
  -> Reconstruction directly from tables and better functions are in work

The current Likelihood function: The current SPE or MPE reconstruction do not use the full information (or are even wrong). With help of TWR the full waveforms can be evaluated.

-> Work underway on all 3 fields
Transient Waveform Recording (TWR)

100 MHz FADC, 12 bit, (Struck)
Installed ~580 channels 2002/03
Complex waveforms

Very complex structures of multi-photon hits not measured by the old DAQ
Need for pattern recognition and appropriate likelihood models -> in work
Advantages from TWR

• Dead-time: 10-20 % -> 0%

• No limitation on the number of Hits (max 8 for the TDC)

• Larger dynamic range

• Measurement of the amplitude of each pulse and photon counting

• In ice each sensor is a full muon detector
  For very high energy events individual PMTs sample hundreds of photons.
  The arrival time distribution strongly depends on the distance.
  Measurement of \(d\) and \(E\) -> in each sensor (2 parameter fit)
• Simulation of old DAQ in TWR data leads to identical results (not shown)
• How to test new methods: Experimental test
  • Sort hits by time and split in 2 independent samples (odd - even)
  • Angular difference is a measure of the angular resolution
    (but worse than the full sample)
Present Reconstruction has been proven to be sufficient to produce good initial physics results:

- Background rejection up to $10^{-8}$
- High efficiency up to the horizon (→ point source analysis)
- Angular resolution: ~0.8° (IceCube), ~2° (AMANDA-II)

Large room for improvements:

- Optical model: Depth dependence, Parameters
- Likelihood model: SPE, MPE, pulse trains ...
- Physics model: Hadr. Vertex, brems cascades, Bundles, Starting/Stopping $\mu$,
- Pattern recognition, iterative methods

Transient Waveform Recording:

- Pattern recognition, each PMT is a detector
- Accurate photon timing, MPE Likelihood, Amplitude likelihood
- No Dead-time
- Larger dynamic range, improved reconstruction at large energies